

Homework #2

Exercise 1. Denote $\langle x \rangle := (1 + |x|^2)^{1/2}$. Let $s \in \mathbb{R}$. Prove that $\int_{\mathbb{R}^n} \langle x \rangle^{-s} dx < +\infty$ if and only if $s > n$.

Exercise 2. Let $p \geq 1$. Prove the following inclusions

$$\mathcal{S}(\mathbb{R}^n) \subset L^p(\mathbb{R}) \subset \mathcal{S}'(\mathbb{R}).$$

Exercise 3. Let $x_0 \in \mathbb{R}^n$. Define $\delta_{x_0}(\varphi) := \varphi(x_0)$ for $\varphi \in \mathcal{S}(\mathbb{R}^n)$. Prove that δ_{x_0} is a tempered distribution and compute its Fourier transform.