Homework #1

Exercise 1. Assume $f \in L^2(\Omega)$. Proof that there exists a unique weak solution in $H^1(\Omega)$ to the following PDE:

$$\begin{cases} -\Delta u = f & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega. \end{cases}$$

Exercise 2. Assume $g \in H^{1/2}(\partial\Omega)$. Proof that there exists a unique weak solution in $H^1(\Omega)$ to the following PDE:

$$\begin{cases} -\Delta u = 0 & \text{in } \Omega, \\ u = g & \text{on } \partial\Omega. \end{cases}$$

Exercise 3. Calculate $(\partial_t^2 - \Delta)\Big(A(t,x)e^{ik\phi(t,x)}\Big)$, and arrange the result in descending order with respect to the powers of k.