

**Homework #1** (due March 14<sup>th</sup>)

*Exercise 1.* Assume  $f \in L^2(\Omega)$ . Proof that there exists a unique weak solution in  $H^1(\Omega)$  to the following PDE:

$$\begin{cases} -\Delta u = f & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega. \end{cases}$$

*Exercise 2.* Assume  $g \in H^{1/2}(\partial\Omega)$ . Proof that there exists a unique weak solution in  $H^1(\Omega)$  to the following PDE:

$$\begin{cases} -\Delta u = 0 & \text{in } \Omega, \\ u = g & \text{on } \partial\Omega. \end{cases}$$

*Exercise 3.* Calculate  $(\partial_t^2 - \Delta)\left(A(t, x)e^{ik\phi(t, x)}\right)$ , and arrange the result in descending order with respect to the powers of  $k$ .